

CH - 2

POLYNOMIALS.

- A polynomial $p(x)$ in one variable x is an algebraic expression of the form -

$$p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \dots + a_1 x + a_0$$

$a_0, a_1, a_2, \dots, a_{n-1}, a_n = \text{constants}$
 $a_n \neq 0, n = \text{positive integer}$

Each of $a_n x^n, a_{n-1} x^{n-1}, \dots, a_0$ is called Term of the polynomial.

- Degree of Polynomial - Highest power of variable x in a polynomial $p(x)$.

- Polynomials on the basis of number of terms -

Monomial $\rightarrow$ Polynomial having one term.
eg:- $2x, 5y^2, 7y^3$.

Binomial $\rightarrow$ Polynomial having 2 terms.
eg:- $x+2, x^2-9$.

Trinomial $\rightarrow$ Polynomial having 3 terms.
eg:- $x^2+2xy+y^2, x+y+z$.

- Polynomials on the basis of degree

Constant Polynomial → Polynomial of degree 0.
eg:- $p(x) = 3$
 $q(x) = -7$.

Linear Polynomial → Polynomial of degree 1.
eg:- $p(x) = x + 3$
 $q(x) = 2x - 5$.

Quadratic Polynomial → Polynomial of degree 2.
eg:- $p(x) = x^2 + 6x + 9$
 $q(x) = 4x^2 - 20x + 25$.

Cubic Polynomial → Polynomial of degree 3.
eg:- $x^3 + 9x^2 + 27x + 81$.

- Zero of Polynomial -

A real number 'a' is zero of a polynomial $p(x)$ if $p(a) = 0$.

a = root of equation $p(x)$.

- Remainder Theorem -

If $p(x)$ is any polynomial of degree greater than or equal to 1, $p(x)$ is divided by the linear polynomial $(x - a)$, then the remainder is $p(a)$, where a is any real number.

• Factor Theorem -

If $p(x)$ is a polynomial of degree $n \geq 1$ & a is any real number, then

- (i) $(x-a)$ is a factor of $p(x)$, if $p(a)=0$
- (ii) $p(a)=0$, if $(x-a)$ is a factor of $p(x)$.

Ex $\rightarrow$ 2.2.

Q4) Find the zero of the polynomial -

(i) $p(x) = x+5$.
solⁿ - $p(x) = x+5$.

$\Rightarrow x+5=0$

$\Rightarrow \boxed{x = -5}$

(vii) $p(x) = cx+d$, $c \neq 0$, $c, d \rightarrow$ real numbers.

$p(x) = cx+d$.

$\Rightarrow cx+d=0$

$\Rightarrow \boxed{x = -\frac{d}{c}}$

Ex $\rightarrow$ 2.3

Q3) $7+3x=0$.

$\Rightarrow 3x = -7$.

$\Rightarrow x = \frac{-7}{3}$.

Remainder = $3\left(\frac{-7}{3}\right)^3 + 7\left(\frac{-7}{3}\right)$

$$= \frac{-343}{9} + \frac{-49}{3}$$

$$= \frac{-490}{9} \neq 0$$

$\therefore 7+3x$ is not a factor of $3x^3+7x$

Ex $\rightarrow$ 2.4

Q3) (i) If $x-1$ is a factor of $p(x)$,
then $p(1) = 0$.

$$\Rightarrow (1)^2 + 1 + K = 0 \quad \text{By Factor Theorem}$$

$$\Rightarrow 2 + K = 0$$

$$\Rightarrow \boxed{K = -2}$$

Q4) (i) $12x^2 - 7x + 1$.

Using middle term split,

$$= (12x^2 - 4x) - 3x + 1$$

$$= (12x^2 - 4x) - (3x - 1)$$

$$= 4x(3x - 1) - 1(3x - 1)$$

$$= (4x - 1)(3x - 1)$$

Q5) (i) $x^3 - 2x^2 - x + 2$.

factors of 2 are ± 1 & ± 2

By trial method, we find $p(1) = 0$

so, $(x-1)$ is a factor of $p(x)$.

$$p(-1) = 0.$$

So, $(x+1)$ is a factor of $p(x)$

Also, by trial method,

$$p(-2) = 0$$

$\therefore (x+2)$ is a factor of $p(x)$.

$$x^3 - 2x^2 - x + 2 = (x+1)(x-1)(x+2).$$

Ex - 2.5.

Q14(i) $(-12)^3 + (7)^3 + (5)^3$.

let, $a = -12$

$b = 7$

$c = 5$.

If $x+y+z=0$, then $x^3+y^3+z^3=3xyz$.

$$-12 + 7 + 5 = 0$$

$$\therefore (-12)^3 + (7)^3 + (5)^3 = 3 \times (-12) \times 7 \times 5 \\ = -1260.$$

(ii) $(+28)^3 + (-15)^3 + (-13)^3$.

let, $a = 28$

$b = -15$

$c = -13$.

$$x+y+z = 28 + (-15) + (-13) = 0$$

If $x+y+z=0$, then $x^3+y^3+z^3=3xyz$.

$$(28)^3 + (-15)^3 + (-13)^3 = 3 \times (28) \times (-15) \times (-13) \\ = 16380.$$

————— x —————

ASSIGNMENT

DEBOLINA GHOSH

Part-A

1. Write the coefficient of y^3 in $5y^3 + 2y^2 - y + 5$
2. Find the coefficient of x^2 in $(x^2 - 1)(x - 2)$
3. If $(x - 2)$ is one of the factor of $3x - 2a$, then find the value of a .
4. Find the degree of polynomial $\frac{x^3 + 3x - 1}{5} - \frac{5}{2}x^2 - x^5$
5. If $p(x) = x^3 - 3x^2 + 2x - 3$ find the value of $p(1) + p(-1)$.
6. Find zeros of the polynomial $z^2 - 8$
7. Dividend = Divisor $\times$ Quotient + _____.
8. Give an example of Trinomial of degree 3.
9. Give one example each of monomial, binomial and quadratic polynomial.
10. Check whether $x = 3$ is a zero of polynomial $x^2 - 3x + x - 3$.
11. Write the degree of the polynomial $\sqrt{7}$
12. If one of the zero of polynomial $3x^2 + 5x + k$ is -1 , then find out the value of k .
13. Express $4x^2 - 4x + 1$ as a square of binomial.

Part - B

14. Check whether $q(x)$ is a multiple of $r(x)$ or not.
If $q(x) = 2x^3 - 11x^2 - 4x + 5$, $r(x) = 2x + 1$
15. Show that $(x - 5)$ is a factor of $x^3 - 3x^2 - 4x - 30$.
16. Evaluate by using suitable identity : $(997)^3$

17. Find the zeroes of the polynomial $p(x) = x(x-2)(x+3)$
18. Find the quotient when $3x^2 - 7x - 6$ is divided by $(x-3)$
19. Factorise $8x^3 + \sqrt{27}y^3$.
20. If $p(x) = x + 9$, then find $p(x) + p(-x)$.
21. Find the product without multiplying directly
 106×94
22. IF $36x^2 - b = \left(6x + \frac{1}{5}\right)\left(6x - \frac{1}{5}\right)$ then find the value of b .
23. Expand using suitable identity $(2x - 3y + z)^2$
24. Find the value of $(351)^2 - (350)^2$.

Part - C

25. Factorise : $64a^2 + 96ab + 36b^2$
26. Factorise : $x^3 + 6x^2 + 11x + 6$
27. If $x^2 + y^2 = 49$ and $x - y = 3$, then find the value of $x^3 - y^3$.
28. Simplify : $(5a - 2b)(25a^2 + 10ab + 4b^2) - (2a + 5b)(4a^2 - 10ab + 25b^2)$.
29. Find the sum of remainders when $x^3 - 3x^2 + 4x - 4$ is divided by $(x - 1)$ and $(x + 2)$.
30. Find the product $\left(p - \frac{1}{p}\right)\left(p + \frac{1}{p}\right)\left(p^2 + \frac{1}{p^2}\right)\left(p^4 + \frac{1}{p^4}\right)$
31. Factorise : $7\sqrt{2}k^2 - 10k - 4\sqrt{2}$.
32. Simplify : $(3x - 4y)^3 - (3x + 4y)^3$
33. Expand : $\left(\frac{1}{2}x - \frac{1}{4}y + 2\right)^2$ using suitable identity.
34. Simplify : $(x + y + z)^2 - (x - y - z)^2$.

CHAPTER-2
POLYNOMIALS

ANSWERS

1. 5
2. -2
3. $a=3$
4. 5
5. -12
6. $+\sqrt{8}, -\sqrt{8}$
7. Remainder
8. $x^3 - 3x^2 + 2$ or any other example
9. $2x, 2x^2 + 3, x^2 + 2x - 3$ or any other examples
10. Yes
11. Degree = 0
12. $k=2$
13. $(2x-1)^2$
14. No.
15. Hint put $x=5$
16. 991026973
17. 0, 2, -3
18. $3x+2$
19. $(2x + \sqrt{3}y)(4x^2 - 2\sqrt{3}xy + 3y^2)$
20. 18
21. Hint $(100+6)(100-6)$
22. $\frac{1}{25}$
23. $4x^2 + 9y^2 + z^2 - 12xy - 6yz + 4xz$
24. 701
25. $(8a+6b)^2$
26. $(x+1)(x+2)(x+3)$
27. 207
28. $117a^3 - 133b^3$
29. -34
30. $p^8 - \frac{1}{p^8}$
31. $(k - \sqrt{2})(7\sqrt{2}k + 4)$
32. $-8y(16y^2 + 27x^2)$ or $-128y^3 - 216x^2y$
33. $\frac{x^2}{4} + \frac{y^2}{16} + 4 - \frac{1}{4}xy - y + 2x$
34. $4xy + 4zx$